Lectures 5:
Design Theory Part I

Today's Lecture

1. Normal forms & functional dependencies
   - ACTIVITY: Finding FDs

2. Finding functional dependencies

3. Closures, superkeys & keys
   - ACTIVITY: The key or a key?

1. Normal forms & functional dependencies

What you will learn about in this section

1. Overview of design theory & normal forms
2. Data anomalies & constraints
3. Functional dependencies
4. ACTIVITY: Finding FDs

Design Theory

- Design theory is about how to represent your data to avoid ***anomalies***.

- It is a mostly mechanical process
  - Tools can carry out routine portions

- *We have a notebook implementing all algorithms!*
  - *We'll play with it in the activities!*

Normal Forms

- 1st Normal Form (1NF) = All tables are flat
- *2nd Normal Form = disused*
- **Boyce-Codd Normal Form (BCNF)**
- **3rd Normal Form (3NF)**

DB designs based on *functional dependencies*, intended to prevent data ***anomalies***

- *4th and 5th Normal Forms = see text books*

Our focus in this lecture + next one

1^{st} Normal Form (1NF)

Student	Courses
Mary	{CS145,CS229}
Joe	{CS145,CS106}
…	…

Violates 1NF.

Student	Courses
Mary	CS145
Mary	CS229
Joe	CS145
Joe	CS106

In 1^{st} NF

1NF Constraint: Types must be atomic!

Data Anomalies & Constraints

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes ***anomalies***:

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..	..	..

If every course is in only one room, contains ***redundant*** information!

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes ***anomalies***:

Student	Course	Room
Mary	CS145	B01
Joe	CS145	C12
Sam	CS145	B01
..	..	..

If we update the room number for one tuple, we get inconsistent data = an ***update*** **anomaly**

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes ***anomalies***:

Student	Course	Room
..	..	..

If everyone drops the class, we lose what room the class is in! = a ***delete* anomaly**

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes ***anomalies***:

...	CS229	C12

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..	..	..

Similarly, we can't reserve a room without students = an ***insert* anomaly**

Constraints Prevent (some) Anomalies in the Data

Student	Course
Mary	CS145
Joe	CS145
Sam	CS145
..	..

Course	Room
CS145	B01
CS229	C12

Is this form better?

- Redundancy?
- Update anomaly?
- Delete anomaly?
- Insert anomaly?

Today: develop theory to understand why this design may be better **and** how to find this *decomposition*…

Functional Dependencies

Functional Dependency

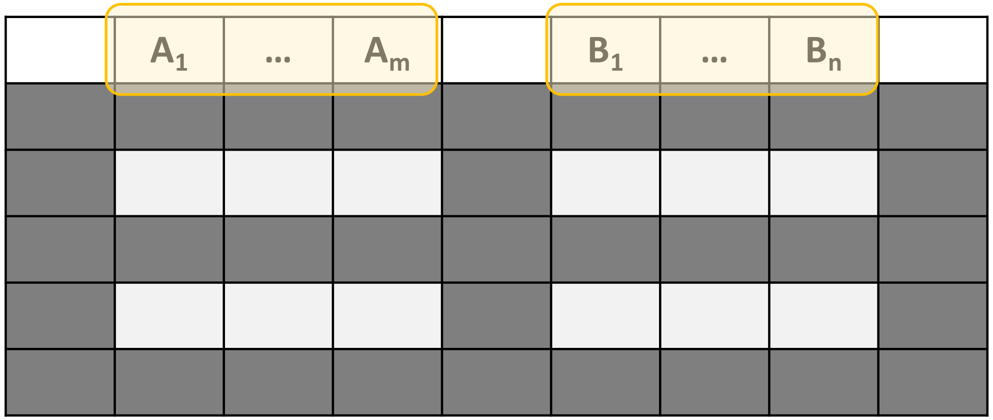
Def: Let A,B be *sets* of attributes
We write A → B or say A ***functionally determines*** B if, for any tuples t_1 and t_2:

$$t_1[A] = t_2[A] \text{ implies } t_1[B] = t_2[B]$$

and we call A → B a functional dependency

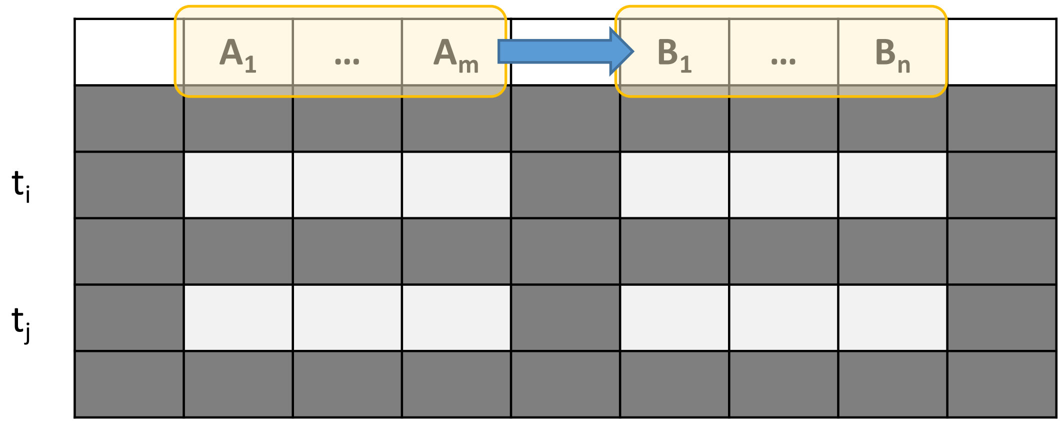
A->B means that
"whenever two tuples agree on A then they agree on B."

A Picture Of FDs



<u>Defn (again):</u>
Given attribute sets $A=\{A_1,\ldots,A_m\}$ and $B = \{B_1,\ldots B_n\}$ in R,

A Picture Of FDs

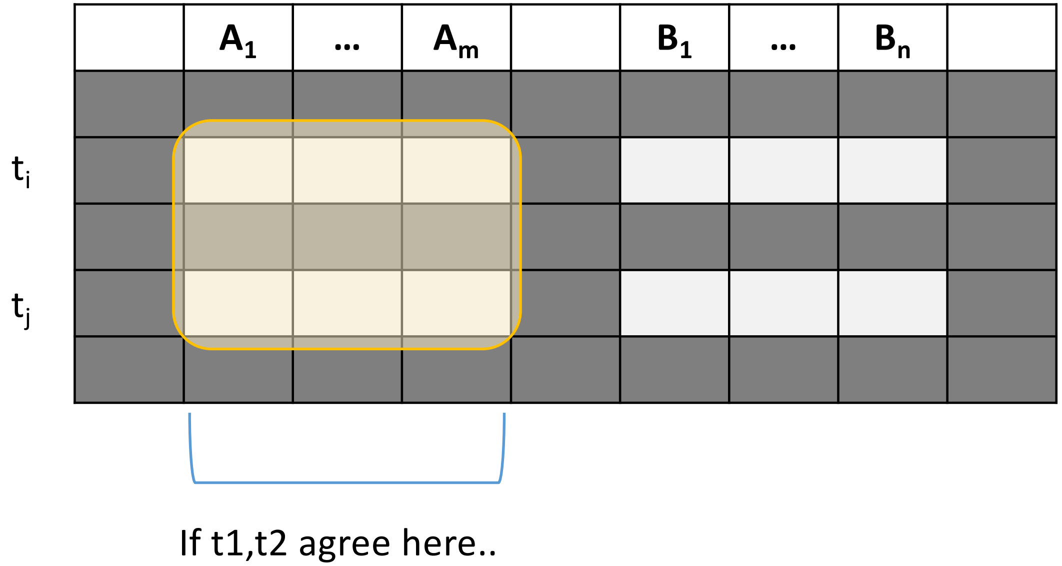


Defn (again):

Given attribute sets $A=\{A_1,\ldots,A_m\}$ and $B = \{B_1,\ldots B_n\}$ in R,

The *functional dependency* A→ B on R holds if for ***any*** t_i,t_j in R:

A Picture Of FDs

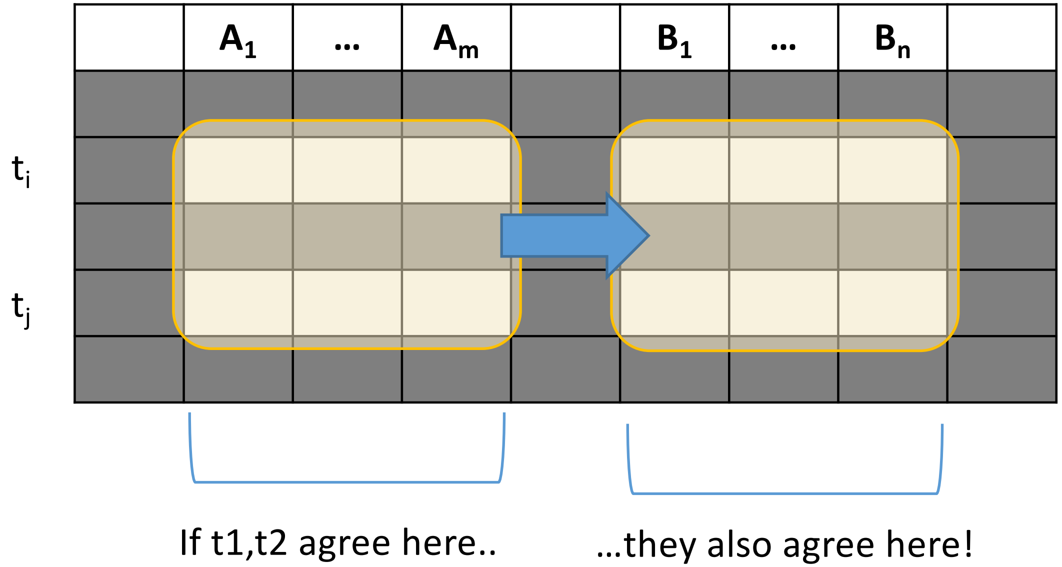


<u>Defn (again):</u>
Given attribute sets $A=\{A_1,\ldots,A_m\}$ and $B = \{B_1,\ldots B_n\}$ in R,

The *functional dependency* $A \rightarrow B$ on R holds if for ***any*** t_i,t_j in R:

<u>if</u> $t_i[A_1] = t_j[A_1]$ AND $t_i[A_2]=t_j[A_2]$ AND … AND $t_i[A_m] = t_j[A_m]$

A Picture Of FDs



<u>Defn (again):</u>
Given attribute sets **A={A_1,…,A_m}** and **B = {B_1,…B_n}** in **R,**

The *functional dependency* A→ B on **R** holds if for ***any*** t_i,t_j in R:

<u>if</u> $t_i[A_1] = t_j[A_1]$ AND $t_i[A_2]=t_j[A_2]$ AND … AND $t_i[A_m] = t_j[A_m]$

<u>then</u> $t_i[B_1] = t_j[B_1]$ AND $t_i[B_2]=t_j[B_2]$ AND … AND $t_i[B_n] = t_j[B_n]$

FDs for Relational Schema Design

- High-level idea: **why do we care about FDs?**

  1. Start with some relational *schema*

  2. Find out its *functional dependencies (FDs)*

  3. Use these to *design a better schema*
     1. One which minimizes the possibility of anomalies

Functional Dependencies as Constraints

A **functional dependency** is a form of **constraint**

- *Holds* on some instances (but not others) – can check whether there are violations
- Part of the schema, helps define a *valid* instance

*Recall: an __instance__ of a schema is a multiset of tuples conforming to that schema, **i.e. a table***

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..	..	..

Note: The FD {Course} -> {Room} ***holds on this instance***

Functional Dependencies as Constraints

Note that:

- You can check if an FD is **violated** by examining a single instance;
- However, you **cannot prove** that an FD is part of the schema by examining a single instance.
  - *This would require checking every valid instance*

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..	..	..

However, cannot *prove* that the FD {Course} -> {Room} is ***part of the schema***

More Examples

An FD is a constraint which holds, or does not hold on an instance:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

More Examples

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

{Position} → {Phone}

More Examples

EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

but *not* {Phone} → {Position}

ACTIVITY

A	B	C	D	E
1	2	4	3	6
3	2	5	1	8
1	4	4	5	7
1	2	4	3	6
3	2	5	1	8

Find at least *three* FDs which are violated on this instance:

{ } → { }
{ } → { }
{ } → { }

2. Finding functional dependencies

What you will learn about in this section

1. “Good” vs. “Bad” FDs: Intuition

2. Finding FDs

3. Closures

4. ACTIVITY: Compute the closures

"Good" vs. "Bad" FDs

We can start to develop a notion of **good** vs. **bad** FDs:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

Intuitively:

EmpID -> Name, Phone, Position is *"good FD"*

- ***Minimal redundancy, less possibility of anomalies***

“Good” vs. “Bad” FDs

We can start to develop a notion of **good** vs. **bad** FDs:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

Intuitively:

EmpID -> Name, Phone, Position is *“good FD”*

But Position -> Phone *is a “bad FD”*

- ***Redundancy! Possibility of data anomalies***

“Good” vs. “Bad” FDs

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..	..	..

Returning to our original example… can you see how the “bad FD” {Course} -> {Room} could lead to an:

- Update Anomaly
- Insert Anomaly
- Delete Anomaly
- …

Given a set of FDs (from user) our goal is to:

1. Find all FDs, and
2. Eliminate the “Bad Ones".

FDs for Relational Schema Design

- High-level idea: **why do we care about FDs?**

1. Start with some relational *schema*
2. Find out its *functional dependencies (FDs)*
3. Use these to *design a better schema*
    1. One which minimizes possibility of anomalies

This part can be tricky!

Finding Functional Dependencies

- There can be a very **large number** of FDs…
  - *How to find them all efficiently?*

- We can't necessarily show that any FD will hold **on all instances…**
  - *How to do this?*

We will start with this problem:
Given a set of FDs, F, what other FDs ***must*** hold?

Finding Functional Dependencies

Equivalent to asking: Given a set of FDs, F = $\{f_1,\ldots f_n\}$, does an FD g hold?

Inference problem: How do we decide?

Finding Functional Dependencies

Example:

Products

Name	Color	Category	Dep	Price
Gizmo	Green	Gadget	Toys	49
Widget	Black	Gadget	Toys	59
Gizmo	Green	Whatsit	Garden	99

Provided FDs:

1. {Name} → {Color}
2. {Category} → {Department}
3. {Color, Category} → {Price}

Given the provided FDs, we can see that {Name, Category} → {Price} must also hold on **any instance**…

Which / how many other FDs do?!?

Finding Functional Dependencies

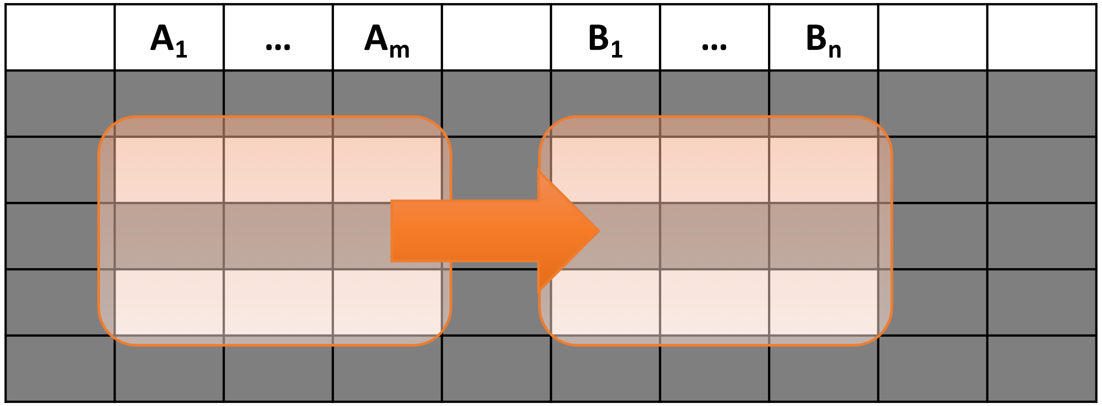
Equivalent to asking: Given a set of FDs, F = $\{f_1,\ldots f_n\}$, does an FD g hold?

Inference problem: How do we decide?

Answer: Three simple rules called **Armstrong's Rules.**

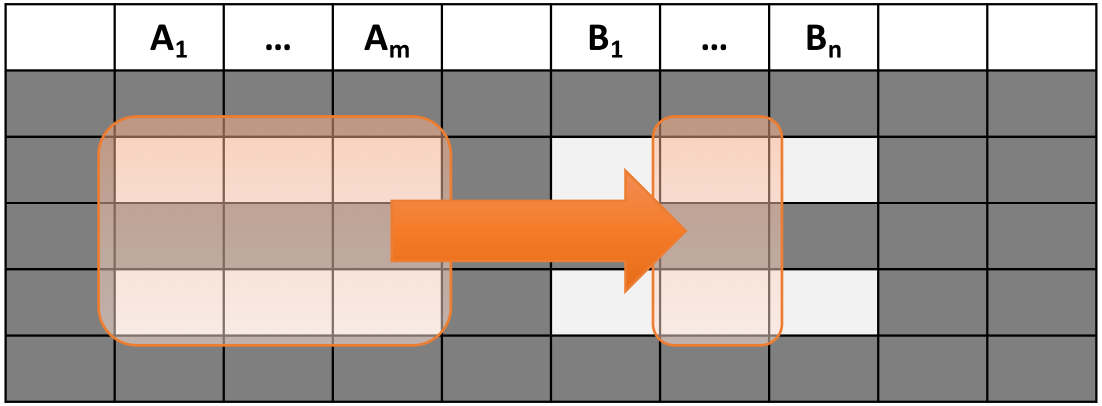
1. **Split/Combine,**
2. **Reduction, and**
3. **Transitivity…** *ideas by picture*

1. Split/Combine



$A_1, \ldots, A_m \rightarrow B_1, \ldots, B_n$

1. Split/Combine

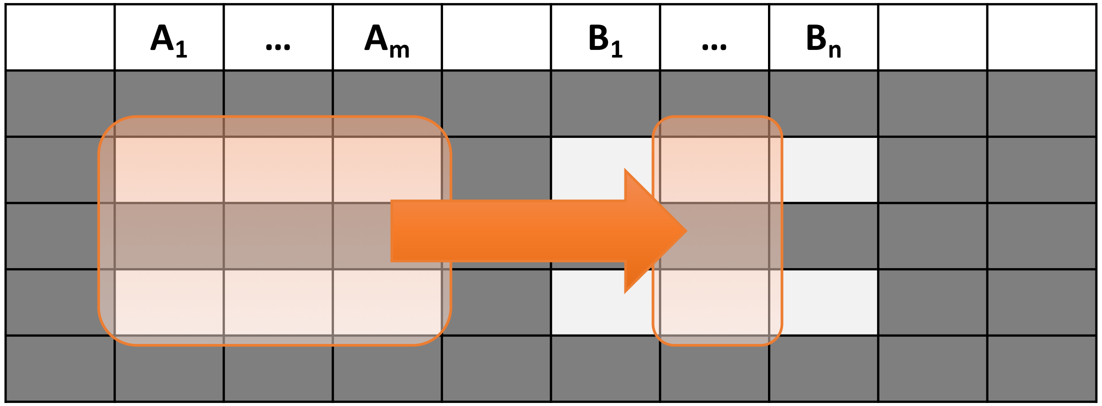


$A_1, \ldots, A_m \rightarrow B_1,\ldots,B_n$

… is equivalent to the following *n* FDs…

$A_1,\ldots,A_m \rightarrow B_i$ for $i=1,\ldots,n$

1. Split/Combine

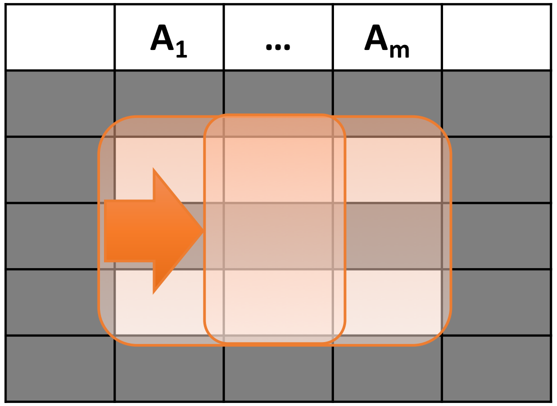


And vice-versa, $A_1,\ldots,A_m \rightarrow B_i$ for i=1,…,n

… is equivalent to …

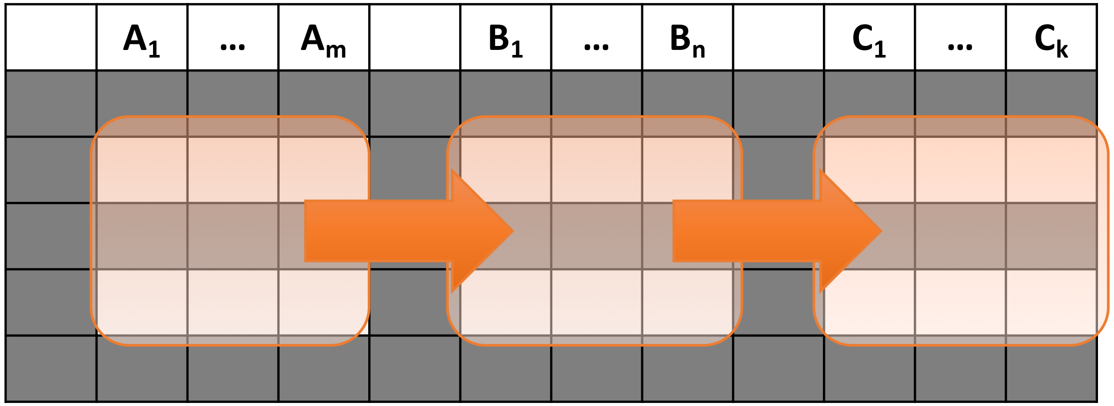
$A_1, \ldots, A_m \rightarrow B_1,\ldots,B_n$

2. Reduction/Trivial



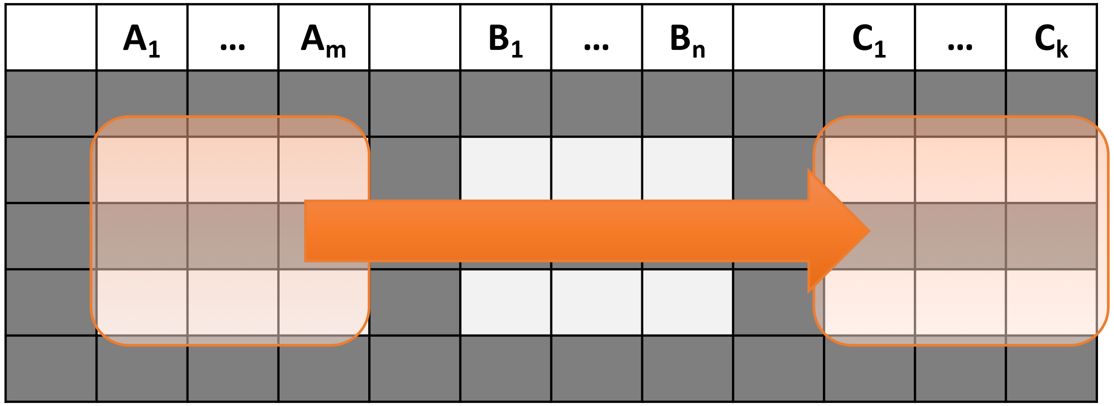
$A_1,\ldots,A_m \rightarrow A_j$ for any $j=1,\ldots,m$

3. Transitive Closure



$A_1, \ldots, A_m \rightarrow B_1,\ldots,B_n$ and
$B_1,\ldots,B_n \rightarrow C_1,\ldots,C_k$

3. Transitive Closure



$A_1, \ldots, A_m \rightarrow B_1,\ldots,B_n$ and
$B_1,\ldots,B_n \rightarrow C_1,\ldots,C_k$

implies

$A_1,\ldots,A_m \rightarrow C_1,\ldots,C_k$

Finding Functional Dependencies

Example:

Products

Name	Color	Category	Dep	Price
Gizmo	Green	Gadget	Toys	49
Widget	Black	Gadget	Toys	59
Gizmo	Green	Whatsit	Garden	99

Provided FDs:

1. {Name} → {Color}
2. {Category} → {Department}
3. {Color, Category} → {Price}

Which / how many other FDs hold?

Finding Functional Dependencies

Example:

Inferred FDs:

Inferred FD	Rule used
4. {Name, Category} -> {Name}	?
5. {Name, Category} -> {Color}	?
6. {Name, Category} -> {Category}	?
7. {Name, Category -> {Color, Category}	?
8. {Name, Category} -> {Price}	?

Provided FDs:

1. {Name} → {Color}
2. {Category} → {Dept.}
3. {Color, Category} → {Price}

Which / how many other FDs hold?

Finding Functional Dependencies

<u>Example:</u>

Inferred FDs:

Inferred FD	Rule used
4. {Name, Category} -> {Name}	Trivial
5. {Name, Category} -> {Color}	Transitive (4 -> 1)
6. {Name, Category} -> {Category}	Trivial
7. {Name, Category -> {Color, Category}	Split/combine (5 + 6)
8. {Name, Category} -> {Price}	Transitive (7 -> 3)

Provided FDs:

1. {Name} → {Color}
2. {Category} → {Dept.}
3. {Color, Category} → {Price}

Can we find an algorithmic way to do this?

Closures

Closure of a set of Attributes

Given a set of attributes $A_1, \ldots, A_n$ and a set of FDs **F**:
Then the **closure**, $\{A_1, \ldots, A_n\}^+$ is the set of attributes B s.t. $\{A_1, \ldots, A_n\} \rightarrow B$

Example: F =

```
{name} → {color}
{category} → {department}
{color, category} → {price}
```

Example Closures:

```
{name}+ = {name, color}
{name, category}+ =
{name, category, color, dept, price}
{color}+ = {color}
```

Closure Algorithm

Start with X = $\{A_1, \ldots, A_n\}$ and set of FDs F.

Repeat until X doesn't change; **do**:

if $\{B_1, \ldots, B_n\} \rightarrow C$ is entailed by F

and $\{B_1, \ldots, B_n\} \subseteq X$

then add C to X.

Return X as X^+

Closure Algorithm

Start with X = $\{A_1, \ldots, A_n\}$, FDs F.
Repeat until X doesn't change; **do**:
  if $\{B_1, \ldots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \ldots, B_n\} \subseteq X$:
    then add C to X.
Return X as X^+

```
{name, category}+ =
                  {name, category}
```

F =

```
{name} → {color}

{category} → {dept}

{color, category} →
                  {price}
```

Closure Algorithm

Start with X = {A_1, …, A_n}, FDs F.
Repeat until X doesn't change; **do**:
if {B_1, …, B_n} → C is in F **and** {B_1, …, B_n} ⊆ X:
then add C to X.
Return X as X^+

```
{name, category}+ =
{name, category}
```

```
{name, category}+ =
        {name, category, color}
```

F =

```
{name} → {color}

{category} → {dept}

{color, category} →
                {price}
```

Closure Algorithm

Start with X = {A_1, …, A_n}, FDs F.
Repeat until X doesn't change; **do**:
if {B_1, …, B_n} → C is in F **and** {B_1, …, B_n} ⊆ X:
then add C to X.
Return X as X^+

F =

```
{name} → {color}

{category} → {dept}

{color, category} →
                {price}
```

```
{name, category}+ =
{name, category}
```

```
{name, category}+ =
{name, category, color}
```

```
{name, category}+ =
   {name, category, color, dept}
```

Closure Algorithm

Start with X = $\{A_1, \ldots, A_n\}$, FDs F.
Repeat until X doesn't change; **do**:
if $\{B_1, \ldots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \ldots, B_n\} \subseteq X$:
then add C to X.
Return X as X^+

F =

{name} → {color}

{category} → {dept}

{color, category} → {price}

$\{name, category\}^+ =$ {name, category}

$\{name, category\}^+ =$ {name, category, color}

$\{name, category\}^+ =$ {name, category, color, dept}

$\{name, category\}^+ =$ {name, category, color, dept, price}

Example

```
R(A,B,C,D,E,F)
```

```
{A,B} → {C}
{A,D} → {E}
{B} → {D}
{A,F} → {B}
```

Compute $\{A,B\}^+ = \{A, B, \qquad \}$

Compute $\{A, F\}^+ = \{A, F, \qquad \}$

Example

`R(A,B,C,D,E,F)`

```
{A,B} → {C}
{A,D} → {E}
{B} → {D}
{A,F} → {B}
```

Compute $\{A,B\}^+$ = {A, B, C, D }

Compute $\{A, F\}^+$ = {A, F, B }

Example

```
R(A,B,C,D,E,F)
```

```
{A,B} → {C}
{A,D} → {E}
{B} → {D}
{A,F} → {B}
```

Compute $\{A,B\}^+$ = {A, B, C, D, E}

Compute $\{A, F\}^+$ = {A, B, C, D, E, F}

3. Closures, Superkeys & Keys

What you will learn about in this section

1. Closures Pt. II
2. Superkeys & Keys
3. ACTIVITY: The key or a key?

Why Do We Need the Closure?

- With closure we can find all FD's easily

- To check if $X \rightarrow A$
  1. Compute X^+
  2. Check if $A \in X^+$

Note here that **X** is a *set* of attributes, but **A** is a *single* attribute. Why does considering FDs of this form suffice?

Recall the **Split/combine** rule:
$X \rightarrow A_1, \ldots, X \rightarrow A_n$
implies
$X \rightarrow \{A_1, \ldots, A_n\}$

Using Closure to Infer ALL FDs

Example:

Given F =

$\{A,B\} \rightarrow C$
$\{A,D\} \rightarrow B$
$\{B\} \rightarrow D$

Step 1: Compute X^+, for every set of attributes X:

$\{A\}^+ = \{A\}$
$\{B\}^+ = \{B,D\}$
$\{C\}^+ = \{C\}$
$\{D\}^+ = \{D\}$
$\{A,B\}^+ = \{A,B,C,D\}$
$\{A,C\}^+ = \{A,C\}$
$\{A,D\}^+ = \{A,B,C,D\}$
$\{A,B,C\}^+ = \{A,B,D\}^+ = \{A,C,D\}^+ = \{A,B,C,D\}$
$\{B,C,D\}^+ = \{B,C,D\}$
$\{A,B,C,D\}^+ = \{A,B,C,D\}$

No need to compute all of these- why?

Using Closure to Infer ALL FDs

Example:

Given F =

{A,B}	$\rightarrow$	C
{A,D}	$\rightarrow$	B
{B}	$\rightarrow$	D

Step 1: Compute X^+, for every set of attributes X:

$\{A\}^+ = \{A\}$, $\{B\}^+ = \{B,D\}$, $\{C\}^+ = \{C\}$, $\{D\}^+ = \{D\}$, $\{A,B\}^+ = \{A,B,C,D\}$, $\{A,C\}^+ = \{A,C\}$, $\{A,D\}^+ = \{A,B,C,D\}$, $\{A,B,C\}^+ = \{A,B,D\}^+ = \{A,C,D\}^+ = \{A,B,C,D\}$, $\{B,C,D\}^+ = \{B,C,D\}$, $\{A,B,C,D\}^+ = \{A,B,C,D\}$

Step 2: Enumerate all FDs $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \varnothing$:

$\{A,B\} \rightarrow \{C,D\}$, $\{A,D\} \rightarrow \{B,C\}$, $\{A,B,C\} \rightarrow \{D\}$, $\{A,B,D\} \rightarrow \{C\}$, $\{A,C,D\} \rightarrow \{B\}$

Using Closure to Infer ALL FDs

<u>Example:</u>

Given F =

```
{A,B} → C
{A,D} → B
{B}   → D
```

Step 1: Compute X^+, for every set of attributes X:

```
{A}+ = {A}, {B}+ = {B,D}, {C}+ = {C}, {D}+ =
{D}, {A,B}+ = {A,B,C,D}, {A,C}+ = {A,C},
{A,D}+ = {A,B,C,D}, {A,B,C}+ = {A,B,D}+ =
{A,C,D}+ = {A,B,C,D}, {B,C,D}+ = {B,C,D},
{A,B,C,D}+ = {A,B,C,D}
```

Step 2: Enumerate all FDs X → Y, s.t. $Y \subseteq X^+$ and $X \cap Y = \varnothing$:

"Y is in the closure of X"

```
{A,B} → {C,D}, {A,D} → {B,C},
{A,B,C} → {D}, {A,B,D} → {C},
{A,C,D} → {B}
```

Using Closure to Infer ALL FDs

Example:

Given F =

{A,B}	→	C
{A,D}	→	B
{B}	→	D

Step 1: Compute X^+, for every set of attributes X:

{A}⁺ = {A}, {B}⁺ = {B,D}, {C}⁺ = {C}, {D}⁺ = {D}, {A,B}⁺ = {A,B,C,D}, {A,C}⁺ = {A,C}, {A,D}⁺ = {A,B,C,D}, {A,B,C}⁺ = {A,B,D}⁺ = {A,C,D}⁺ = {A,B,C,D}, {B,C,D}⁺ = {B,C,D}, {A,B,C,D}⁺ = {A,B,C,D}

Step 2: Enumerate all FDs X → Y, s.t. $Y \subseteq X^+$ and $X \cap Y = \emptyset$:

{A,B} → {C,D}, {A,D} → {B,C}, {A,B,C} → {D}, {A,B,D} → {C}, {A,C,D} → {B}

The FD X → Y is non-trivial

Superkeys and Keys

Keys and Superkeys

A <u>superkey</u> is a set of attributes $A_1, \ldots, A_n$ s.t.
for *any other* attribute **B** in R,
we have $\{A_1, \ldots, A_n\} \rightarrow B$

i.e. all attributes are *functionally determined* by a superkey

A <u>key</u> is a *minimal* superkey

This means that no subset of a key is also a superkey (i.e., dropping any attribute from the key makes it no longer a superkey)

Finding Keys and Superkeys

- For each set of attributes X

  1. Compute X^+

  2. If X^+ = set of all attributes then X is a **superkey**

  3. If X is minimal, then it is a **key**

Example of Finding Keys

```
Product(name, price, category, color)
```

```
{name, category} → price
{category} → color
```

What is a key?

Example of Keys

```
Product(name, price, category, color)
```

```
{name, category} → price
{category} → color
```

{name, category}$^{+}$ = {name, price, category, color}

= the set of all attributes

⟹ this is a **superkey**

⟹ this is a **key**, since neither name nor category alone is a superkey

Activity-5-1.ipynb

Summary

- Constraints allow one to reason about **redundancy** in the data

- Normal forms describe how to **remove** this redundancy by **decomposing** relations
  - Elegant—by representing data appropriately certain errors are essentially impossible
  - For FDs is the normal form.